Exercise 11

For the following exercises, sketch the parametric equations by eliminating the parameter. Indicate any asymptotes of the graph.

$$x = 6\sin(2\theta), \quad y = 4\cos(2\theta)$$

Solution

Multiply both sides of the equation for x by $2/3$.

$$\frac{2}{3}x = 4\sin(2\theta), \quad y = 4\cos(2\theta)$$

Divide both sides of each equation by 4.

$$\frac{x}{6} = \sin(2\theta), \quad \frac{y}{4} = \cos(2\theta)$$

Square both sides of each equation and add the respective sides together.

$$\left(\frac{x}{6}\right)^2 + \left(\frac{y}{4}\right)^2 = \sin^2(2\theta) + \cos^2(2\theta)$$

Simplify both sides.

$$\frac{x^2}{36} + \frac{y^2}{16} = 1$$

This is an ellipse with major axis in the x-direction and minor axis in the y-direction. Below is a plot of the parametric equations as t goes from 0 to π.